MANIFESTATION OF THE ENERGY CONSERVATION LAW IN THE METRIC TENSOR OF THE EXPANDING UNIVERSE

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We performed a comparative analysis of the expressions for the kinetic and potential energies of the bodies corresponding to the metric tensor of the standard cosmological model with and without consideration of the dependence of the scale factor of the expanding Universe from time.

We have shown that the use of the metric that does not consider the change of the scale factor leads to the exclusion from the analysis of the kinetic and potential energy of the bodies, and the metric that considers the variability of the scale factor adds them to the analysis and gives us a simple expression of the mechanical energy conservation law for any material object in the Universe.

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1. Introduction. Reflection of the kinetic and potential energy in the corresponding system of coordinates and metric tensor of the standard cosmological model.

Einstein in "The basics of the general relativity theory" [1] points out several times that the equations of the general relativity theory represent a mathematical expression of the full energy conservation law that includes the energy of the matter and of the gravitational field.

With this, the standard cosmological model that was built after on the basis of the Einstein's relativity theory studies the material objects of the Universe in the corresponding system of coordinates which reflects in the equations in the form of zero velocities of the material objects [2]. The necessity of the choice of this system of coordinates is, as it is known, related to the condition of the isotropy of the space that is equivalent to the absence of the distinguished directions which performs at the zero module of the velocity vectors. But satisfying to the condition of the isotropy, the corresponding coordinates exclude from the energetic member of the equations the kinetic energy of the expansion without compensating it by the corresponding changes in the other members. This way the kinetic energy of the expansion of the Universe happens to be excluded from the analysis.

The metric that is in the basis of the standard model [2], is expressed by the correlation:

$$ds^2 = c^2dt^2 - a^2[d\chi^2 + \sin^2\chi\,(\sin^2\theta\, d\varphi^2 + d\theta^2)]; \qquad (1)$$

where a — is a radius of the curve of the space (scale factor), χ — a coordinate of the range, θ, φ — angular coordinates, c - lightspeed. Corresponding values of the components of the metric tensor: $g_{00} = 1$, $g_{11} = - a^2$, $g_{22} = - a^2 \sin^2\chi$, $g_{33} = - a^2 \sin^2\chi \sin^2\theta$.

Expression (1) corresponds to the uniformly curved space obtained by Einstein by adding an imaginary 4th spatial coordinate [3] and its further exclusion via the radius of the curve of the space. His mathematical formalism that allows to describe the curve of the 3-dimensional space by the gravitational fields, was introduced by Einstein when he examined the stationary Universe. When deriving the formula (1), the differential of the 4th spatial coordinate that is the part of the expression for the element of the spatial distance dl, includes the differentials of the three other spatial coordinates and does not include the differential of the radius of the curve of the space da that is equal to zero in the stationary Universe [2].

$$dl^2 = dx_1^2 + dx_2^2 + dx_3^2 + (x_1\, dx_1 + x_2\, dx_2 + x_3\, dx_3)^2 / (a^2 - x_1^2 - x_2^2 - x_3^2). \qquad (2)$$

Let us examine the relation of the metric (1) to the value of the gravitational potential φ, that defines the value of the potential energy U_g of the body with the mass m in a gravitational field.

By the definition of the gravitational potential

$$U_g = m\varphi. \qquad (3)$$

From the equation of the general relativity theory is follows [2], that is case of the relatively low velocities of the movement of the matter the gravitational potential is related to g_{00} by the expression:

$$g_{00} = (1 + 2\varphi/c^2). \qquad (4)$$

But in (1) $g_{00} = 1$, from where it follows that the metric in the basis of the standard cosmological model corresponds to the zero values of the potential energy of the material objects in the gravitational field of the Universe. This way the potential energy of the bodies in the gravitational field of the Universe in the standard model also appears to be excluded from the analysis.

2. Manifestation of the energy conservation law with consideration of the non-zero value of the scale factor differential in the expanding Universe.

Let us see how the expression for the interval (1) will change when we take into consideration an obvious dependence of the scale factor from the time a(t). Introducing, in a similar way to [1],[2], the notion about 4-dimensional space, we will obtain the expression for the element of the spatial distance dl in the form:

$$dl^2 = dx_1^2 + dx_2^2 + dx_3^2 + (a\, da - x_1\, dx_1 - x_2\, dx_2 - x_3\, dx_3)^2 / (a^2 - x_1^2 - x_2^2 - x_3^2); \quad (5)$$

where x_1, x_2, x_3 are the Cartesian space coordinates. Going from the Cartesian coordinates to the polar r, θ, φ and examining for simplicity only the radial movements ($\theta = 0$, $d\theta = 0$), we obtain

$$dl^2 = dr^2 + (da - (r/a)*dr)^2 / (1 - (r/a)^2); \qquad (6)$$

let us introduce in a similar way to [2], the coordinate χ from the expression $r = \sin(\chi)$. Then:

$$dl^2 = da^2 + a^2 d\chi^2; \qquad (7)$$

Let us note the expression for the interval:

$$ds^2 = c^2dt^2 - dl^2 = c^2dt^2 - da^2 - a^2d\chi^2 = c^2dt^2(1 - da^2/c^2dt^2) - a^2d\chi^2; \qquad (8)$$

Denoting $da/cdt = a'$, we will finally obtain:

$$ds^2 = c^2dt^2(1 - a'^2) - a^2d\chi^2; \qquad (9)$$

This way taking into consideration the obvious dependence a(t) gives the expression for the interval where the constant value of the component of the metric tensor $g_{00} = 1$ is changed to the variable

$$g_{00} = (1-a'^2). \qquad (10)$$

Inserting to (1), we finally note down:

$$ds^2 = c^2dt^2(1-a'^2) - a^2[d\chi^2 + \sin^2\chi\,(\sin^2\theta\, d\varphi^2 + d\theta^2)]. \qquad (11)$$

Closed Universe in a standard cosmological model represents an expanding -dimensional hypersphere in a 4-dimensional Euclidean [4] space, the radius of which is increasing with the velocity $V = da/dt$. In this space the notions of the length (the radius of the curve of the space) and time are defined, and this, at the consequential applying of this formalism introduced by Einstein, requires the propagation of the notions of the velocity and the kinetic energy and also of the notion of the maximal speed (lightspeed), the Einstein's relativity principle and the corresponding expressions of the special relativity theory.

Comparing (3) and (9), we obtain

$$a'^2 = -2\varphi/c^2; \qquad (12)$$

$$\varphi = -(1/2)a'^2/c^2; \qquad (13)$$

Considering that $a' = da/cdt$

$$\varphi = -(1/2)(da/dt)^2. \qquad (14)$$

Value (da/dt) is the velocity of the expansion of the hypersphere with which all the material objects of the Universe move in a 4-dimensional space in a direction that is perpendicular to all the coordinate axises of the 3-dimensional space. By denoting this velocity as V_4, considering (2) we obtain:

$$U_g = -\, m\, V_4^2/2. \qquad (15)$$

But $mv^2/2$ is a non-relativistic (for the cases of the low velocities, one of which is the expression (4) for the gravitational potential) expression for the kinetic energy E_k of the movement of the body with the mass m (in this case the movement in a 4-dimensional space). From where

$$U_g = -\, E_k. \qquad (16)$$

Expression (16) represents the energy conservation law that applies equally for all the material objects of the Universe that are on the surface of the hypersphere and have the same velocity V_4. In the point of the maximal expansion when $E_k = 0$, U_g also turns to 0, and in other moments of the time the potential energy of the body in the gravitational field is negative and numerically equal to its kinetic energy of the movement in the 4-dimensional space with the opposite sign. The full mechanical energy of the body W_m in any moment of the time is equal to zero:

$$W_m = U_g + E_k = 0, \qquad (17)$$

which expresses the energy conservation law that represents in this formulation the law of the development of the closed Universe.

It is natural to expect that this law will be kept at high velocities of the movement, i.e. far from the point of the maximal expansion of the Universe but with the transition to the more complicated relativistic expressions for the kinetic and potential energy.

Let us note that it does not require to refuse the corresponding system of the coordinates that provides the requirement of the isotropy of the space as the movement happens in a direction that is perpendicular to all coordinate axises of the 3-dimensional space and the velocities of the objects in this space is still considered to be equal to zero.

3. Conclusion

The use in the standard model of the expanding Universe of the metric that does not consider the dependence of the scale factor from time leads to the exclusion from the analysis of the kinetic and potential energy of the bodies. The consideration of this dependence introduces it to the analysis and gives a simple expression for the energy conservation law that relates to any material object of the Universe.

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