Electron model in the form of four-dimensional ball in Minkowski space

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Abstract. In solving one mechanical problem the author faced the necessity to take into account the presence of free electrons in a macroscopic body. Specific character of this problem did not permit to use the traditional Dirac's model of the electron. At the same time, it required that a model of the electron had been Lorentz-invariant.

In Minkowski space, three geometric objects are Lorentz-invariant: a point, light cone, and pseudosphere. Point model of electric charge leads to a singularity of the self-electromagnetic field of charge at the location of charge. In addition, this model does not allow us to calculate the self-action of charge and it has other known lacks.

Therefore, two variants of model of electron were considered: the light cone and pseudosphere (both having Lorentz-invariant inner structures). The first variant of the model correctly describes the electromagnetic field of an arbitrarily moving charge, moreover this field turns out a purely geometric effect and its calculation does not require the use of Maxwell's equations. The second variant of the model, namely pseudosphere with inner structure, permits to calculate the electron spin on the basis of usual rules of mechanics. In this variant, the anomalous magnetic moment of electron, i.e. a small difference between g-factor and the integer 2, is explained by self-action of electron; in addition, the fine structure constant receives a new definition. One may hope that the constructed model of the electron will be useful in solving some physical problems.

1. Introduction

In solving one mechanical problem the author faced the necessity to take into account the presence of free electrons in a macroscopic body.

The properties of electron are described with a pinpoint accuracy in the quantum electrodynamics. P. A. M. Dirac was the first who proposed relativistic model of the electron [1]. Further, the theory of the electron was continued to improve (see, for example, [2, 3, 4]). All known properties of electron systematized in the review [5].

Specific character of the mentioned mechanical problem demanded that the proper angular momentum (spin) of electron been computed by usual rules of mechanics, but not be a consequence of fulfilment of the Dirac equation, as it occurs in quantum electrodynamics. This difficulty was overcome due to following circumstance. In the solution of the mentioned mechanical problem the final aim was to find some macroscopic characteristics of the considered body. In this case there was no need to specify state of electrons in such full measure as quantum theory does by means of a wave function. In particular, the required accuracy of describing the motion of electrons was such that, for example, the trace, which a moving electron leaves in a cloud chamber, can be regarded as a geometric (i. e., infinitely thin) line. Bearing in mind this circumstance it became possible to donate full description of electron state and construct a model of the electron on a basis of usual rules of mechanics. In addition, it was required that the symmetry of a model of the electron was described by the Lorentz group. Taking into account these requirements, *four-dimensional (pseudo)ball in Minkowski space, endowed with a certain structure, is regarded as a model of the electron.*

The model is constructed in two stages. In the first stage the light cone, uniformly filled with timelike straight lines, is taken as a geometric image of the electron. In the second stage timelike straight lines are replaced by some helical objects. An outer boundary of such construction is a (pseudo)sphere, therefore in this model the electron can be considered as a four-dimensional ball in Minkowski space. This geometric image of the electron is called a *particle*.

This model has following properties. It describes the electric and magnetic fields of an arbitrarily moving charge (herewith an electromagnetic field is a purely geometric effect). A finiteness of particle radius allows one to compute the spin of particle by usual rules of mechanics and to compute the self-action of particle.

The developed model does not give such complete description of a state of electron as the quantum electrodynamics does. But one may hope that this model will be useful not only for the solution of concrete mechanical problem, but also for the solution of some physical problems.

2. A "point" particle

In constructing a theory of elementary particles, it is natural to use as the initial manifold the Minkowski space being a foundation of special relativity.

This space, denoted by M, is four-dimensional real pseudo-Euclidean space of signature $(+ - - -)$.

In the works [6, 7] it was suggested a model, in which an electron is represented in the space M as a family of timelike straight lines, called *rays*, which are intersected at a single point, called a *center*, and uniformly fill the interior of a light cone with vertex at this point. The center moves in the space M along a smooth timelike line L called a *world line*. During the movement the angles between all rays and the angles between each ray and a tangent to the world line remain unchanged. The rays are regarded as geometric images of certain material objects; their points are called *elements*. This geometric construction is called a *particle* (Fig. 1).

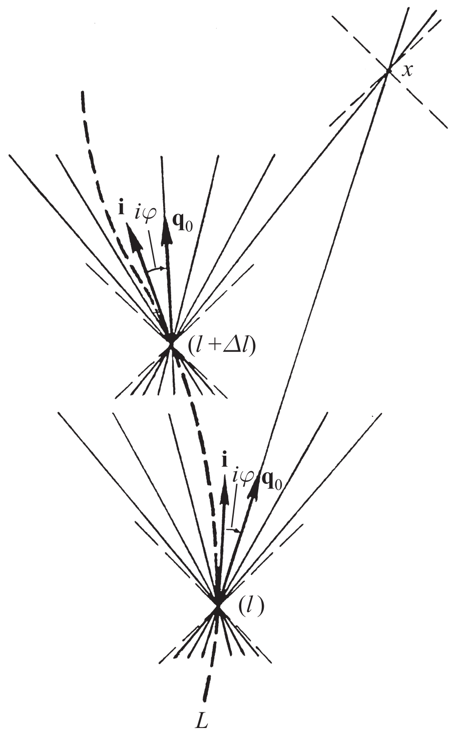

Fig. 1. Two positions of particle on the world line.

The angles $i\varphi$ between the unit vector $\mathbf{i}$ tangent to the world line L and the directing unit vectors of rays $\mathbf{q}_0$ are not changed when the particle moves (the fine dashed lines are light cones; i is the imaginary unit; l is a natural parameter on L; x is an arbitrary point of Minkowski space M).

In the case of a particle of finite radius the straight-lines, passing through a center of particle, are rays axes; $\mathbf{q}_0$ is the directing unit vector of ray axis.

Each element of ray is assigned two-valent antisymmetric tensor (bivector) $\mathbf{q}_0\mathbf{j}_0 - \mathbf{j}_0\mathbf{q}_0$, where $\mathbf{q}_0$ is the directing unit vector of the ray ($\mathbf{q}_0$ is oriented to the future), $\mathbf{j}_0$ is a velocity of element with respect to the space M (the product of vectors is tensorial).

By definition, the velocity $\mathbf{j}_0$ of ray element is a derivative of displacement vector of element with respect to the natural parameter l on the world line L.

In the work [6] it is shown that a ray element, which is situated at the distance Q from the center of particle, moves with the velocity

$$\mathbf{j}_0 = \mathbf{i} + \left(\frac{d\mathbf{i}}{dl}\,\mathbf{i} - \mathbf{i}\,\frac{d\mathbf{i}}{dl}\right) \cdot \mathbf{Q}. \tag{1}$$

Here $\mathbf{i}$ is a tangent unit vector to the world line L with the origin at the center of particle and directed toward the future; $\mathbf{Q} = \pm Q\mathbf{q}_0$ is a vector connecting the center of particle with the considered element of ray (upper and lower signs refer to the cases when the ray element is situated in the future and in the past relative to the center of particle, respectively); $\mathbf{q}_0$ is the directing unit vector of ray, oriented to the future; the tensor product of vectors is denoted without sign multiplication between them; the dot denotes scalar multiplication. The velocity $\mathbf{j}_0$ is a dimensionless quantity. If for the description of particle motion it is necessary to use a value of velocity dimension, then in place of the value $\mathbf{j}_0$ it is taken $c\mathbf{j}_0$ ($c\mathbf{j}_0$ is a derivative of displacement vector of element with respect to the proper time of particle $t = l/c$; c is the speed of light).

In the right-hand side of (1) the first summand describes a translational component of particle motion and the second one a hyperbolic rotation, of particle, due to which the angle between the vectors $\mathbf{q}_0$ and $\mathbf{i}$ remains constant during particle motion. In this case the first summand is independent of the distance Q between the center of particle and the element under consideration and the second summand is proportional to Q. Note that the hyperbolic rotation is also called a boost.

Since a particle moves along the world line, the rays of particle from its various locations on the world line pass through each fixed point x of space M (see Fig. 1). The integration of the mentioned above bivector over all such particle locations with the weight proportional to the number of rays in the neighborhood of x gives the following *tensor of self-electromagnetic field of particle* F_s [6]:

$$\begin{aligned} F_s(x) &= a \int\limits_{(L_-)} \frac{\mathbf{q}_0\mathbf{j}_0 - \mathbf{j}_0\mathbf{q}_0}{S(Q)}\, dl = \\ &= \left.\frac{e(\mathbf{Q}\mathbf{i} - \mathbf{i}\mathbf{Q})}{(\mathbf{Q}\cdot\mathbf{i})^3}\right|_* + \left.\frac{e\left[(\mathbf{Q}\cdot\mathbf{i})\left(\mathbf{Q}\dfrac{d\mathbf{i}}{dl} - \dfrac{d\mathbf{i}}{dl}\mathbf{Q}\right) - \left(\mathbf{Q}\cdot\dfrac{d\mathbf{i}}{dl}\right)(\mathbf{Q}\mathbf{i} - \mathbf{i}\mathbf{Q})\right]}{(\mathbf{Q}\cdot\mathbf{i})^3}\right|_* . \end{aligned} \tag{2}$$

Here a is a scalar parameter proportional to a total number of rays and is assumed to be equal to $4\pi e$; e is the charge of electron; L_- is a part of particle world line, which is inside the past light cone of point x; Q is four-dimensional interval (a distance) between the center of particle and the point x; $S(Q)$ is an area of the part of pseudosphere of radius Q centered at the center of particle, which is inside particle's future light cone; $\mathbf{q}_0$ is the directing unit vector of the ray that passes through the point x; $\mathbf{j}_0$ is a velocity of ray element situated at the point x (the velocity $\mathbf{j}_0$ is given by formula (1), at which the vector $\mathbf{Q}$ has the end at the point x and the origin at the particle center moving along L, $\mathbf{Q} = Q\mathbf{q}_0$); the asterisk belongs to all functions in the expression, marked by it, and means that each function is evaluated at the point y of intersection of the world line L and the past light cone of point x, for example, $\mathbf{Q}_* = \overrightarrow{yx}$ ($\mathbf{Q}_*$ is an isotropic vector). Integral in (2) is taken over the part L_- of the world line in order to satisfy the principle of causality, according to which the physical characteristics at a point x may depend only on the objects situated inside the past light cone of this point. Note that a singularity, arising in computing the integral, is compensated by a singularity of a pseudosphere area $S(Q)$.

The obtained value of tensor of self-electromagnetic field of particle F_s leads to the expressions for electric and magnetic fields of particle that are identical (up to notation) with well-known expressions for electric and magnetic fields of point charge. These fields are usually computed with provision for Maxwell equations and Lienard-Wiechert potentials [8]. It should be remarked that here in computing the field F_s the Maxwell equations are not applied and a matching scalar parameter a is used only. In this model an electromagnetic field is a purely geometric effect.

As noted above, in formula (1) the first summand is independent of the distance Q and the second summand is proportional to Q. This implies that at the right-hand side of formula (2) in a nonrelativistic approximation the first and the second terms are proportional to $1/r^2$ and $1/r$ respectively (r is a distance between the center of particle and the point x in three-dimensional physical space). The second term is a wave component of electromagnetic field. In the case of rectilinear world line (in uniform moving a particle), the second term vanishes because in this case $d\mathbf{i}/dl = \mathbf{0}$.

The interpenetration of rays of different particles results in that *the principle of superposition of electromagnetic fields is satisfied.*

In the considered model, the particle has zero size in three-dimensional physical space concomitant to its center, therefore such particle called a *"point" particle* [6, 7].

3. A particle of finite radius

Let us perfect the model in such a way that it describes the *spin* of the electron.

We shall say, as before, that a particle is a geometric object in Minkowski space, which is a certain family of rays. However now we shall suppose that each ray is a certain three-dimensional helical object, with timelike axis, which consists of n_* identical "thick" helical lines and rotates about its axis (Fig. 2). Below we shall give a detailed description of this construction.

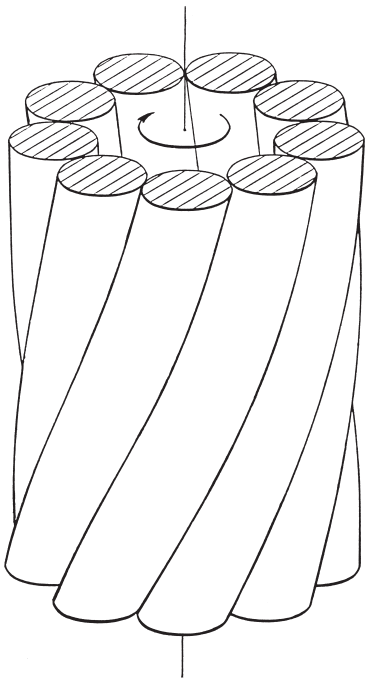

Fig. 2. One of rays of particle.

First we give the definition of rays axes. We shall call the *axes* of rays of particle the timelike straight lines that are intersected at a single point, namely at the *center* of particle, and are distributed uniformly over the interior of light cone with a vertex at this point (see Fig. 1). We shall call all family of rays axes a *skeleton* of particle.

We introduce now a notion of ray. Let $\mathbf{i}$ be a timelike unit vector (it will play the role of a tangent unit vector to the world line of particle). We denote by $\mathbf{q}_0$ the directing unit vector of axis of arbitrary ray. By assumption, both vectors are directed toward the future and have their origins at the particle center.

Introduce for each axis of ray three-dimensional affine subspace $C_{\mathbf{q}_0} \subset M$ such that it contains this axis and its associated vector space is as follows

$$Lin\{\mathbf{q}_0\} \oplus Lin^{\perp}\{\mathbf{i},\ \mathbf{q}_0\} \quad (\text{for } \mathbf{q}_0 \neq \mathbf{i}),$$

where the symbol $Lin\{\mathbf{x},\ \mathbf{y},\ ...\}$ denotes a linear hull of vectors $\mathbf{x},\ \mathbf{y},\ ...$; $\perp$ is a passage to orthogonal complement in a vector space associated with M; $\oplus$ is a direct sum of vector spaces. Further, we shall denote an affine space and the associated with it vector space by the same symbol.

In the plane $Lin^{\perp}\{\mathbf{i},\ \mathbf{q}_0\} \subset C_{\mathbf{q}_0}$ we give a figure consisting of n_* identical disks of radius p_*, which contact to each other and the centers of which are situated along a circle of radius r_* (Fig. 3). The figure shows that

$$p_* = r_* \sin \frac{\pi}{n_*}. \tag{3}$$

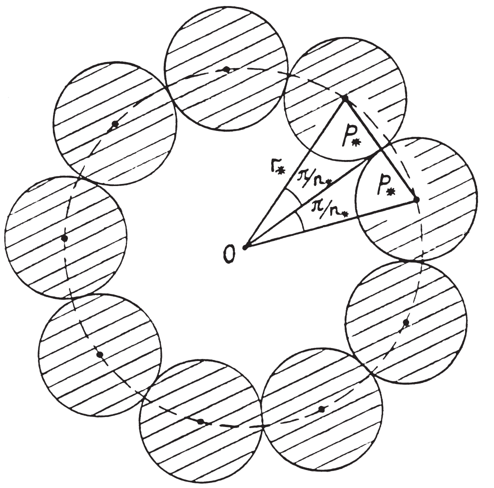

Fig. 3. Cross-section of ray
(the number of disks shown is conventional).

We shall say that a *ray* with the axis $Lin\{\mathbf{q}_0\}$ is three-dimensional helical object in $C_{\mathbf{q}_0}$, which this figure "sweeps out" in the case of helical motion in $C_{\mathbf{q}_0}$ along the axis $Lin\{\mathbf{q}_0\}$. In such a motion of this figure each its point "sweeps out" in $C_{\mathbf{q}_0}$ a helical line, which will be called a *thread.* The equation of thread is a usual equation of helical line in three-dimensional subspace $C_{\mathbf{q}_0}$.

A family of the threads that are generated by all points of one disk is the mentioned above "thick" helical line. We shall call it a *helix*. The ray consists of n_* identical helices (see Fig. 2). We assume that all rays rotate about their axes with identical and constant angular velocity o (which is defined as a derivative of rotation angle with respect to the proper time of particle).

A ***particle*** *is a family of rays introduced above.* The movement and rotation of rays in the space M occur in such a way that a construction, formed by them, (the particle) repeats itself at all moments of time. Therefore, the particle is undeformable four-dimensional object in Minkowski space M. Let a world line of particle's center is a *world line* of particle.

We shall say that a *proper physical space* of particle is a three-dimensional spacelike hyperplane $\mathit{\Gamma}_p$, passing through the center of particle and orthogonal to the vector $\mathbf{i}$: $\mathit{\Gamma}_p = Lin^{\perp}\{\mathbf{i}\}$. The cross-section of particle by the hyperplane $\mathit{\Gamma}_p$ will be called a *central section* of particle.

If (in rough approximation) the figure in Fig. 3 is regarded as a circular ring of width $2p_*$, then the central section of particle may be imagined in $\mathit{\Gamma}_p$ as a spherical shell of middle radius r_* and half-thickness p_*. It may be shown that in such a rough approximation the outer boundary of particle is a (pseudo)sphere of radius $r_* + p_*$ centered at the center of particle. The equation of this outer boundary has the form $\mathbf{R} \cdot \mathbf{R} = -(r_* + p_*)^2$ (where $\mathbf{R}$ is a radius-vector of sphere points with respect to the center of particle). This is a ruled hypersurface similar to a hyperboloid of one sheet in three-dimensional properly Euclidean space (Fig. 4). This hypersurface has isotropic rulings and isotropic (light) asymptotic cone. Since the outer boundary of particle is close to sphere, the particle itself can be regarded as a four-dimensional *ball* in the space M. Its radius is equal to r_*+p_*. However since, as will be shown, $p_* \ll r_*$, the quantity r_* is called a *radius* of particle.

We note that the validity of using a model of the electron in a form of ball is indirectly confirmed by the work [9], in which it is shown that an electron has a spherical symmetry.

Now we can introduce the notion of particle spin.

The angular momentum $\overline{K}_*$ of ray can be computed as an angular momentum of the figure in Fig. 3, rotating about its center with angular velocity o (provided that each of n_* disks, forming the figure, has a mass equal to helix mass m_*). By (3) and the definition of angular momentum we have

$$\left|\overline{K}_*\right| = n_* m_* \left|o\right| r_*^2 \left[1 + \frac{1}{2}\left(\frac{p_*}{r_*}\right)^2\right] = n_* m_* \left|o\right| r_*^2 \left[1 + \frac{1}{2}\sin^2\left(\frac{\pi}{n_*}\right)\right]. \quad (4)$$

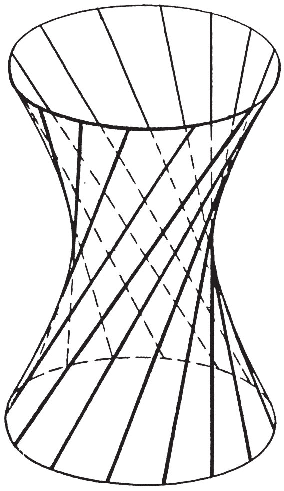

Fig. 4. A hyperboloid of one sheet.

We shall call the pseudovector $\overline{K}_*$ a *spin of ray*. The integration of the spin of ray $\overline{K}_*$ over all rays gives a *spin of particle* $\overline{K}$.

Since the space M is four-dimensional, a rotation of ray about its axis can occur as inside three-dimensional subspace $C_{\mathbf{q}_0}$ as with an intermediate output from the subspace $C_{\mathbf{q}_0}$. One can prove that in the case when a ray rotates about its axis by the angle π with the intermediate output from $C_{\mathbf{q}_0}$ the helixes of ray change their orientation, namely the right-oriented helixes become the left-oriented ones and vice versa. We shall call the process of reorientation of helixes of ray a *reorientation of ray*. Pay attention that in the case when the reorientation of rays occurs the skeleton of particle (consisting of the axes of rays) remains immovable.

It can be proved that the spin of particle $\overline{K}$ is equal to zero in case when the helixes of all rays have the same orientation (in virtue of a spherical symmetry of central section of particle). If the half of rays of particle has helixes of the same orientation and the other half of rays has helixes of opposite orientation, then the spin of particle $\overline{K}$ has a maximum absolute value:

$$\left|\overline{K}\right|_{max} = \frac{1}{2}\, m_e\, |o|\, r_*^2 \left[1 + \frac{1}{2}\,\sin^2\left(\frac{\pi}{n_*}\right)\right], \qquad (5)$$

where, by assumption, $N n_* m_* = m_e$; N is a total number of rays; m_e is a mass of the electron.

In the book [10] the model suggested is considered in more detail.

4. The fine structure constant

By (4) we have

$$\frac{\left|\overline{K}_*\right|}{m_* \left|o\right| r_*^2} = n_* \left[1 + \frac{1}{2}\sin^2\left(\frac{\pi}{n_*}\right)\right] \quad \text{(for } o \neq 0\text{)}. \tag{6}$$

The expression in the right-hand side of (6) is a dimensionless quantity. In the quantum electrodynamics it is known a dimensionless fundamental constant, the fine structure constant α. Let us equate the right-hand side of relation (6) to α^{-1}. We have

$$n_* \left[1 + \frac{1}{2}\sin^2\left(\frac{\pi}{n_*}\right)\right] = \alpha^{-1}. \tag{7}$$

Substituting in (7) the experimental value of $\alpha^{-1} \approx 137.0359991$ [11], we get

$$n_* = 137 \tag{8}$$

with a relative error $1 \cdot 10^{-7}$. The fact that for the quantity n_*, which gives the number of helixes in ray, we obtained the integer value to high precision is one of significant arguments that the model is valid.

Further we shall regard relation (8) as a postulate of this model (Fig. 5). In this case formula (7) is a new *definition of fine structure constant* α.

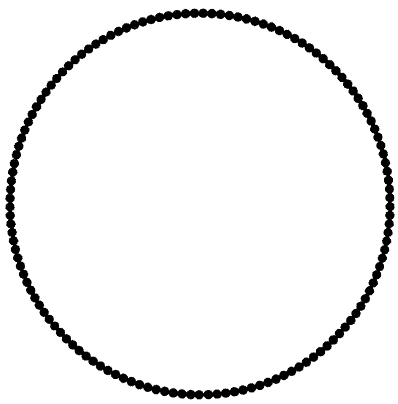

Fig. 5. A cross-section of ray for $n_* = 137$.

From (7) and (8) we have $\alpha \approx 0.0072973518$, what coincides with the experimental value of α equal to 0.0072973526 [11] with a relative error about

$1 \cdot 10^{-7}$. Note that before in the physics the constant α was introduced only as a combination of other fundamental constants: $\alpha = e^2/(\hbar c)$, where e is the charge of the electron, $\hbar$ is the Planck's constant, and c is the speed of light.

From (3) and (8) it follows that $p_*/r_* = \sin(\pi/n_*) \approx 1/44$. Consequently, $p_* \ll r_*$. Therefore the quantity r_* was called before a radius of particle.

5. The laws of particle motion

By assumption, the *equation of motion of thread* has the form

$$\frac{d\mathbf{q}}{dl} = \frac{e}{m_e c^2}\,(F + F_s)\cdot\mathbf{q}. \tag{9}$$

Here all the functions belong to an element (point), of thread, which is located in the central section of particle; $\mathbf{q}$ is a tangent unit vector to the thread, oriented to the future; F is a tensor of external electromagnetic field; F_s is a tensor of self-electromagnetic field of particle; l is a natural parameter on the world line of particle; e and m_e are the charge and the mass of the electron, respectively; c is the speed of light.

The tensor F_s for a particle of finite radius can be computed in the following way [10]. Let us assign the bivector $\mathbf{q}\,\mathbf{j} - \mathbf{j}\,\mathbf{q}$ to each element of thread. Here $\mathbf{q}$ is a unit vector tangent to a thread at this element, $\mathbf{j}$ is an element velocity (which is defined as a derivative of displacement vector of element with respect to the parameter l). Suppose that x is an arbitrary point of space M and the bivector given belongs to a thread element at the point x. The integration of this bivector over all particle locations such that the particle threads pass through x with a weight proportional to a total length of threads in a unit volume of the neighborhood x gives a value of tensor F_s at the point x. The obtained in such a way formula for computing the self-electromagnetic field of particle is similar to formula (2) but is more lengthy. Away from the particle, this field coincides with the field of "point" particle (when averaged over domains of space with linear dimensions exceeding the ray's diameter $2r_*$). Inside the central section of particle at distances less than $r_* - p_*$ from the center of particle the field F_s is zero since the threads in this area is lacking (see Fig. 2 and Fig. 3). Thus, in this model the self-field of charge has no divergence.

Suppose, within the central section of particle the external electromagnetic field F is homogeneous and the average value of $F_s \cdot \mathbf{q}$ is negligible. We integrate both sides of equation (9) over the cross-section of ray, locating in the central section of particle. Since the ray is axisymmetric, the integral of vector $\mathbf{q}$ over the cross-section of ray is proportional to the directing unit vector of ray axis $\mathbf{q}_0$. Therefore under the above conditions on the fields F and F_s this integration gives the following *equation of motion of ray axis*:

$$\frac{d\mathbf{q}_0}{dl} = \frac{e}{m_e c^2}\, F \cdot \mathbf{q}_0, \tag{10}$$

where the value of external electromagnetic field F is taken at the center of particle.

Now we integrate both sides of equation (10) over all rays. In virtue of a spherical symmetry of central section of particle the integral of the vector $\mathbf{q}_0$ over all rays is a vector tangential to the world line. Therefore by this integration we obtain the following *equation of motion of particle center*:

$$\frac{d\mathbf{i}}{dl} = \frac{e}{m_e c^2}\, F \cdot \mathbf{i}. \tag{11}$$

Here $\mathbf{i}$ is a unit vector, tangent to the world line L, directed towards the future and having the origin at the center of particle; a value of the field F is taken, as in (10), at the center of particle.

Equation (11) coincides with the known equation of motion of a point charge in an electromagnetic field F [8]. It describes an acceleration of particle, that is, gives the form of the world line L.

Equation (10) is the new one. It describes a particle acceleration (since it results in equation (11)) and also a rotation of particle, what will be shown later.

In the works [6, 10] for arbitrary electromagnetic field it is shown that a tensor F, a vector of strength of electric field $\mathbf{E}$, and a pseudovector of strength of magnetic field $\overline{H}$ are related as

$$F = \mathbf{E}\boldsymbol{\tau} - \boldsymbol{\tau}\mathbf{E} + \overline{H} \cdot \varkappa; \tag{12}$$

$$\mathbf{E} = F \cdot \boldsymbol{\tau}; \quad \overline{H} = \frac{1}{2} F \cdot\cdot\, \varkappa. \tag{13}$$

Here, by assumption, in Minkowski space M there is given the inertial frame of reference $\{T, \mathit{\Gamma}\}$ with a timelike straight line T (a *time axis*) and the orthogonal to T three-dimensional spacelike hyperplane $\mathit{\Gamma}$ (a *physical space*); $\boldsymbol{\tau}$ is the unit vector of the time axis T, oriented to the future; $\varkappa$ is Levi-Civita pseudotensor over $\mathit{\Gamma}$ (a trivalent absolutely antisymmetric unit pseudotensor); two dots denote a biscalar multiplication.[1] Thus, here $T = Lin\{\boldsymbol{\tau}\}$, $\mathit{\Gamma} = Lin^{\perp}\{\boldsymbol{\tau}\}$, $\mathit{\Gamma} \perp T$. The tensor F is invariant geometric object in the space M. The vector $\mathbf{E}$ and the pseudovector $\overline{H}$ are "components" of tensor F in the frame of reference $\{T, \mathit{\Gamma}\}$ ($\mathbf{E} \in \mathit{\Gamma}$, $\overline{H} \in \mathit{\Gamma}$).

Consider a motion of particle in the neighborhood of a certain instant of its proper time t_0. We introduce the corresponding to this moment instantaneous inertial frame of reference $\{T_p, \mathit{\Gamma}_p\}$ associated with the particle, where $T_p = Lin\{\mathbf{i}_0\}$, $\mathit{\Gamma}_p = Lin^{\perp}\{\mathbf{i}_0\}$, $\mathbf{i}_0$ is a value of the vector $\mathbf{i}$ tangent to the world line at t_0. The straight line T_p and the hyperplane $\mathit{\Gamma}_p$ are instantaneous proper time axis and instantaneous proper physical space of particle, respectively. At time t_0 the center of particle coincides with the point of intersection of T_p and $\mathit{\Gamma}_p$.

Further we shall assume that the strengths $\mathbf{E}$ and $\overline{H}$ of external electromagnetic field belong to the frame of reference $\{T_p, \mathit{\Gamma}_p\}$, and, therefore, they are determined by formulas (13), where $\boldsymbol{\tau} = \mathbf{i}_0$ and $\varkappa$ is a Levi-Civita pseudotensor over $\mathit{\Gamma}_p$. In accordance with the above assumption on the homogeneity of field F, the fields $\mathbf{E}$ and $\overline{H}$ are assumed to be homogeneous inside the central section of particle.

By (11) and (12) we have

$$\frac{d\mathbf{i}}{dl}(t_0) = \frac{e}{m_e c^2}\, \mathbf{E}(t_0), \tag{14}$$

where $\mathbf{E}$ is an electric field strength at the center of particle. Here it is accounted that $\boldsymbol{\tau} = \mathbf{i}_0$, $\mathbf{i}_0 \cdot \mathbf{i}_0 = 1$, $\mathbf{E} \cdot \mathbf{i}_0 = 0$ (because $\mathbf{E} \in \mathit{\Gamma}_p$, $\mathit{\Gamma}_p \perp \mathbf{i}_0$) and $\varkappa \cdot \mathbf{i}_0 = \widehat{0}$ (since $\varkappa$ is a pseudotensor over $\mathit{\Gamma}_p$, $\mathit{\Gamma}_p \perp \mathbf{i}_0$); $\widehat{0}$ is a null bivalent tensor.

From equation (14) it follows that in the case of a homogeneous electromagnetic field F, acting on a particle, the motion of the center of particle depends only on the electric component $\mathbf{E}$ of this field in the instantaneous inertial frame of reference associated with particle.

[1] The rule of scalar and biscalar multiplications of tensors is illustrated by the following example:

$$\mathbf{xyz} \cdot\cdot\, \mathbf{uvw} = \mathbf{xy} \cdot (\mathbf{z} \cdot \mathbf{u})\mathbf{vw} = (\mathbf{z} \cdot \mathbf{u})\mathbf{xy} \cdot \mathbf{vw} = (\mathbf{z} \cdot \mathbf{u})\mathbf{x}(\mathbf{y} \cdot \mathbf{v})\mathbf{w} = (\mathbf{z} \cdot \mathbf{u})(\mathbf{y} \cdot \mathbf{v})\mathbf{xw},$$

where $\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{u}, \mathbf{v}, \mathbf{w}$ are arbitrary vectors; the tensor product is denoted without multiplication sign between multipliers.

If in the central section of particle we have $\mathbf{E} = \mathbf{0}$ at all moments of time ($\mathbf{0}$ is a null vector), then by (14), $\mathbf{i} = \text{const}$ and the world line L is rectilinear. In this case the introduced frame of reference $\{T_p, \mathit{\Gamma}_p\}$ can be regarded as that associated with particle at all moments of time. Here the time axis T_p coincides with L and the physical space $\mathit{\Gamma}_p$ moves translationally, together with the center of particle along T_p, remaining parallel to its position at moment t_0.

Suppose that within the central section of particle the electric field $\mathbf{E}$ is equal to zero at all moments of time and the magnetic field takes a constant value $\overline{H}$. In this case the world line of particle L is rectilinear and, as will be shown below, the particle itself rotates about the axis parallel to $\overline{H}$.

Really, from (10) and (12) we have at $\mathbf{E} = \mathbf{0}$:

$$\frac{d\mathbf{q}_0}{dl} = \frac{e}{m_e c^2}\, \overline{H} \cdot \varkappa \cdot \mathbf{q}_0, \tag{15}$$

where $\overline{H}$ is a strength of magnetic field at the center of particle.

Note that in equation (15) $\varkappa$ is a constant pseudotensor since it is defined over the physical space $\mathit{\Gamma}_p$, which in this case moves translationally along L together with the center of particle and remains parallel to itself. Therefore, in the right-hand side of equation (15) the multiplier of $\mathbf{q}_0$ is a constant.

Represent the vector $\mathbf{q}_0$ as the sum of temporal and spatial components:

$$\mathbf{q}_0 = \mathbf{q}_T + \mathbf{q}_\mathit{\Gamma}, \tag{16}$$

where $\mathbf{q}_T \in T_p$, $\mathbf{q}_\mathit{\Gamma} \in \mathit{\Gamma}_p$ (this decomposition is given by formulas $\mathbf{q}_T = (\mathbf{q}_0 \cdot \mathbf{i}_0)\mathbf{i}_0$ and $\mathbf{q}_\mathit{\Gamma} = \mathbf{q}_0 - \mathbf{q}_T$).

Substituting decomposition (16) in equation (15), we find

$$\frac{d\mathbf{q}_T}{dl} + \frac{d\mathbf{q}_\mathit{\Gamma}}{dl} = -\frac{e}{m_e c^2}\, \overline{H} \times \mathbf{q}_\mathit{\Gamma}. \tag{17}$$

Here it is accounted that $\varkappa \cdot \mathbf{q}_T = \widehat{0}$ (since $\varkappa$ is a pseudotensor over $\mathit{\Gamma}_p$, $\mathbf{q}_T \in T_p$ and $\mathit{\Gamma}_p \perp T_p$) and the equation $\overline{H} \cdot \varkappa \cdot \mathbf{q}_\mathit{\Gamma} = -\overline{H} \times \mathbf{q}_\mathit{\Gamma}$ is used; $\times$ is a sign of vector multiplication in $\mathit{\Gamma}_p$.

Since $d\mathbf{q}_T/dl \in T_p$, $d\mathbf{q}_\mathit{\Gamma}/dl \in \mathit{\Gamma}_p$, $\overline{H} \times \mathbf{q}_\mathit{\Gamma} \in \mathit{\Gamma}_p$, and $\mathit{\Gamma}_p \perp T_p$, from equation (17) it follows that

$$\frac{d\mathbf{q}_T}{dl} = \mathbf{0}; \tag{18}$$

$$\frac{d\mathbf{q}_\Gamma}{dl} = -\frac{e}{m_e c^2}\,\overline{H} \times \mathbf{q}_\Gamma. \tag{19}$$

Let t be the proper time of particle. Taking into account that $dl = c\,dt$ and in the considered case $\mathbf{E} = \mathbf{0}$ (for all t), from (14), (18), and (19) we obtain

$$\frac{d\mathbf{i}}{dt} = \mathbf{0}; \tag{20}$$

$$\frac{d\mathbf{q}_T}{dt} = \mathbf{0}; \tag{21}$$

$$\frac{d\mathbf{q}_\Gamma}{dt} = \overline{\omega} \times \mathbf{q}_\Gamma, \tag{22}$$

where

$$\overline{\omega} = -\frac{e}{m_e c}\,\overline{H} \quad (\overline{\omega} \in \Gamma_p). \tag{23}$$

We recall that equations (20)–(22) refer to the case when inside the central section of particle in the inertial frame of reference associated with particle, the electromagnetic field has null electric component $\mathbf{E}$ and homogeneous constant magnetic component $\overline{H}$. This permits one to make the following conclusions. Equation (20) means that in this case the world line of particle L is rectilinear. Equation (21) implies that the temporal components $\mathbf{q}_T$ of directing vectors $\mathbf{q}_0$ of rays axes are unchanged in time. According to equation (22) the spatial components $\mathbf{q}_\Gamma$ of directing vectors $\mathbf{q}_0$ rotate in Γ_p with the constant angular velocity $\overline{\omega}$ of the form (23), which is the same for all rays.

Thus, from within the physical space Γ_p the particle motion "looks" as a rotation with the angular velocity $\overline{\omega}$ about the axis, which is parallel to $\overline{H}$ and passes through the center of particle. Taking into account that in such a motion of particle in four-dimensional space M the plane $Lin\{\mathbf{i}, \overline{H}\}$ remains immovable, we can say that in the space M the particle rotates about the plane $Lin\{\mathbf{i}, \overline{H}\}$. It should be remarked that in formula (23) the magnetic field $\overline{H}$ must be computed in the inertial frame of reference $\{T_p, \Gamma_p\}$ associated with particle.

In our computations the self-action of particle is neglected since we have ignored the self-electromagnetic field of particle F_s. In the work [10] the magnetic component $\overline{H}_s$ of this field in the case of rotating particle is computed. This magnetic component is proportional to the angular velocity of particle rotation $\overline{\omega}$. Replacing in formula (23) the quantity $\overline{H}$ by the sum $\overline{H} + \overline{H}_s$, in accordance with [10] we find

$$\overline{\omega} = -\,(g/2)\,\frac{e}{m_e c}\,\overline{H} \approx -1{,}0011640\,\frac{e}{m_e c}\,\overline{H}. \tag{24}$$

Here for the denotation of numerical coefficient it is used the symbol $g/2$, where g is the Lande-factor. The comparison of formulas (23) and (24) shows that a fractional part of the coefficient $g/2$ is caused by the *self-action* of particle.

In physics, the formulas of the form (23) and (24) are well known and the coefficient $g/2$ is measured in the experiment with high accuracy. Its experimental value is equal to 1.00115965218073 [5, 11]. The value $g/2 \approx 1.0011640$, obtained from (24), differs from this experimental value by a relative quantity about $4 \cdot 10^{-6}$. In [10] in deriving formula (24) some simplifications were used. Abandoning them, as one might expect, will give the value of $g/2$ with greater precision.

Let us pay attention to the distinction between the interpretations of formula (24) in the present model and in traditional model of the electron. According to the traditional model, this formula describes a precession of the electron spin. In this case for the explanation of the fractional part of coefficient $g/2$ it is introduced the assumption on the existence of virtual particles, which surround an electron and screen partially its charge. In our model formula (24) describes a rotation of particle and, therefore, the fractional part of the coefficient $g/2$ is naturally explained by the influence on the particle its self-electromagnetic field.

6. The magnetic moment of the electron

Consider a motion of particle in inhomogeneous magnetic field.

Suppose, in the frame of reference $\{T_p, \mathit{\Gamma}_p\}$ associated with particle there is a nonzero gradient of magnetic field $\nabla\overline{H}$ inside the central section of particle. In the work [10] it is shown that in such a field the reorientation of the half of particle rays occurs. In this work it is also shown that in this case in virtue of (9) the particle is exposed to the force

$$\mathbf{f} = \nabla(\overline{H} \cdot \overline{\mu})_+ \quad (\,\mathbf{f} \in \mathit{\Gamma}_p). \tag{25}$$

Here

$$\overline{\mu} = \frac{1}{4}\,e\,r_*\overline{k}; \tag{26}$$

r_* is a radius of particle; $\overline{k}$ is a unit pseudovector parallel to a pseudovector of the magnetic field strength $\overline{H}$ (either possible directions of the pseudovector $\overline{k}$, in the line and opposite of direction of $\overline{H}$, is equiprobable in view of equal probability of the reorientation of either half of particle rays under the action of a gradient of magnetic field); in a scalar product of vectors the index $+$ means a change to the positively defined metric form in Γ_p: $(\mathbf{x}\cdot\mathbf{y})_+ = -\mathbf{x}\cdot\mathbf{y}$ for all $\mathbf{x}, \mathbf{y} \in \Gamma_p$ (in which case $(\mathbf{x}\cdot\mathbf{x})_+ \geqslant 0$ for all $\mathbf{x} \in \Gamma_p$). Note that in [10] in the derivation of formula (25) the self-electromagnetic field of particle is not taken into account and it is assumed that the quantities of order p_*/r_* are negligibly small compared with unity (since $p_* \ll r_*$).

In [10] it is also shown that in the considered case the spin $\overline{K}$ of particle is oriented, as well as $\overline{\mu}$, along the pseudovector $\overline{k}$ and its absolute value is given by formula (5).

We assume that a linear velocity of rays rotation is equal to the light velocity:[2]

$$|o|\, r_* = c. \tag{27}$$

In this case by (5) and (27) the spin of particle has the form

$$\overline{K} = \frac{1}{2}\, m_e c\, r_* \overline{k}. \tag{28}$$

Here we have neglected the quantity $(1/2)\sin^2(\pi/n_*)$ compared with unity (since $\sin(\pi/n_*) = p_*/r_* \ll 1$).

By formulas (26) and (28) we get

$$\overline{\mu} = \frac{e}{2m_e c}\, \overline{K}. \tag{29}$$

Note that the directions of $\overline{\mu}$ and $\overline{K}$ are opposite to each other because of a negative sign of the electron charge e.

In the electrodynamics it is known the dependencies similar to dependencies (25) and (29). In these relations the quantities, replacing $\overline{\mu}$ and $\overline{K}$, are

[2] In the framework of the special relativity this assumption is justified. Since Minkowski space is linear, the variables, measured along timelike and spacelike directions, should be measured in the same physical units (or the variables should be dimensionless). For example, as such variables it may be taken natural parameters measured in units of length. In the change to these variables the velocity of objects, having isotropic world lines (that is the light velocity), is equal to unity. Thus, this assumption is means, in fact, that characteristic velocity for the model is assumed to be unity.

a magnetic moment and angular momentum of moving charges, respectively. From the fact that in the considered model the particle spin $\overline{K}$ is the proper angular momentum of particle it follows that the particle possesses the *proper magnetic moment* $\overline{\mu}$ of the form (26).

We emphasize that in this model, the rest energy of the electron $E_0 = m_e c^2$ receives clear mechanical sense. So, in [10] is shown that if relation (27) is satisfied, then the energy E_0 is a kinetic energy of particle with respect to Minkowski space M. The half of this energy is caused by the translational movement of particle along the world line, the other half by the rotation of particle rays about their axes.

A fractional part of the quantity $g/2$, that is, the difference $(g/2) - 1$ (where g is the Lande-factor) is called an anomalous magnetic moment (AMM). For electrons this quantity can be found by formula (24). In the work [12] the effect of the electron anomalous magnetic moment on a differential cross-section of the process of elastic collisions of electrons with atoms of hydrogen was computed. The authors of this work took into account the presence of electric field and laser field with circular polarization. They write: "For the nonrelativistic regime, the addition of the electron's AMM is noticeable but small. When increasing the electric field strength to moderate values, this effect becomes more pronounced" [12].

Note that the cause of a certain varying of differential cross-section may be the following. We remark earlier that in formula (23) the magnetic field $\overline{H}$ must be computed in the inertial frame of reference associated with the particle (not in "immovable" laboratory frame of reference). The same is also valid for (24). The distinction between the values of field $\overline{H}$ in moving and immovable frames of reference becomes significant in the change from a nonrelativistic regime to the relativistic one. This fact may give a significant contribution to the effect discussed in the work [12].

7. Conclusions

The *radiant model of the electron*, which considered above, gives a correct description of main properties of electron.

In particular, it allows one to compute an electromagnetic field of arbitrarily moving charge, and this field has no divergence at the location of charge. In this model, the electron spin is computed by usual rules of mechanics. The anomalous magnetic moment of the electron is explained by self-action of electron. The fine structure constant given a new definition and its value is computed with high precision by means of a simple formula, which follows directly from the laws of mechanics. In this model, the rest energy of electron $E_0 = m_e c^2$ is a kinetic energy of electron with respect to Minkowski space. In fact this model combines in a single object an electric charge and its electromagnetic field, which are usually distinguished. One may hope that this model can be useful in the solution of some physical problems.

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